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# Effects of new leptons in Electroweak Precision Data

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## Abstract

We obtain limits on generic vector-like leptons at the TeV scale from electroweak precision tests. These limits are complementary to the ones obtained from lepton flavour violating processes. In general, the quality of the global electroweak fit is comparable to the one for the Standard Model. In the case of an extra neutrino singlet mixing with the muon or electron, the global fit allows for a relatively large Higgs mass ( $M_H \lesssim 260$  GeV at 90 % C.L.), thus relaxing the tension between the direct LEP limit and the Standard Model fit.

## 1 Introduction

In this paper we analize the effects of new vector-like leptons in electroweak precision data (EWPD). We derive the corresponding effective Lagrangian up to dimension 6, and use it to study these effects from two related points of view. On the one hand, we obtain limits on the couplings and masses of the new particles. On the other, we observe that the new leptons can mildly improve some features of the Standard Model (SM) fit and have consequences

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on the preferred values of the Higgs mass  $M_H$ . The largest limit on  $M_H$  arises for new leptons transforming as neutrino singlets under the SM. If they have Majorana masses, they may act as see-saw messengers and the restrictions from the limits on light neutrino masses must be taken into account.

Let us first review shortly the situation in the SM. As it is well known, EWPD are consistent with the SM to a remarkable precision, sensitive to the details of radiative corrections [1]. Despite this general success, a few experimental results are difficult to accommodate within the SM picture. The discrepancies could have experimental origin, but it is nevertheless interesting to study them in some detail, to see if they follow some pattern and could give us some hint of new physics. The main problem at the Z pole is with the value of  $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ , which is distinctively higher when derived from hadronic asymmetries than when derived from the leptonic ones. The statistical probability that the set of asymmetry data be consistent with the SM hypothesis is only 3.7 % [2]. This low probability is driven by the two most precise determinations of  $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ , obtained from the leptonic asymmetry parameter  $A_l$  by SLD and of the bottom forward-backward asymmetry  $A_{\text{FB}}^{0,b}$  at LEP, respectively. These measurements differ by 3.2 standard deviations ( $\sigma$ ). On the other hand, the SM prediction depends through quantum corrections on the unknown value of the Higgs boson mass  $M_H$ , and agrees with leptonic (hadronic) data for a light (heavy) Higgs. The current global fit in [3] to Z-pole observables plus the masses of the top quark  $m_t$  and W boson  $M_W$ , and the W width  $\Gamma_W$ , prefers a light Higgs:  $M_H = 87^{+36}_{-27} \text{ GeV}$ . Note that this conclusion does not seem compelling, as it arises from the combination of contradictory measurements:  $M_W$  and the leptonic asymmetries at the Z pole point to a very light Higgs, whereas the hadronic asymmetries prefer a heavy Higgs [4]. At any rate, this best-fit value gives a prediction for  $A_{\text{FB}}^{0,b}$  that is  $2.9 \sigma$  above its experimental value, while the leptonic asymmetries differ by less than  $1.6 \sigma$  [3]. For this reason, it is common to speak of a  $A_{\text{FB}}^{0,b}$  anomaly, and implicitly consider that the leptonic data are in good agreement with the SM. One should not forget, however, that LEP 2 has put a sharp limit on the mass of the SM Higgs boson:  $M_H \geq 114.4 \text{ GeV}$  (95% C.L.) [5]. With this constraint, the best SM fit is realized for the lowest allowed  $M_H$ . Then, we find that the pulls in  $A_{\text{FB}}^{0,b}$  and  $A_e(\text{SLD})$  are, respectively, 2.6 and 2.0.

We use a data set including low- $Q^2$  measurements. In Tables 11 and 12 in the appendix, we collect the experimental values of different (pseudo) observ-

ables at different energies, together with the corresponding predictions and pulls in the SM for  $M_H = 114.4$  GeV. We use the new (preliminary) CDF-DØ value for the top mass,  $m_t = 172.6 \pm 1.4$  GeV [6]. We find  $\chi^2/\text{d.o.f.} = 43.9/30$ , which corresponds to a probability of 4.8 % only. More details on this fit are given below. For the moment, let us just point out the main discrepancies between experiment and the SM, beyond the ones stressed above. First, there is a  $2.8\sigma$  discrepancy, coming from the NuTeV experiment, in the effective coupling  $g_L^2$  that enters neutrino-nucleon scattering. Unexpectedly large isospin violations [7] or a significant quark-antiquark asymmetry in the strange sea quarks [8] could account for part of the deviation, but it seems difficult to explain the whole effect with standard physics only. Second, the pulls of  $M_W$  and the hadronic cross section at the Z pole  $\sigma_H^0$  are at  $1.3\sigma$  and  $1.7\sigma$ , respectively. And finally, the data show departures from lepton universality in both  $Z$  and  $W$  decays<sup>1</sup>. There have been several attempts to explain some of these deviations (mainly the  $A_{\text{FB}}^{0,b}$  or NuTeV anomalies) with new physics, see for example [10, 11]. In this regard, it is important to be careful that the new physics that corrects a particular observable does not spoil the goodness of the global fit, and also to have into account the direct LEP 2 lower bound on  $M_H$ .

Here we study the impact that new fermionic  $SU(3)_c$  singlets (leptons) have on EPWD. Since these hypothetical particles modify lepton observables, it looks plausible, a priori, that they may improve the electroweak fit and/or change the prediction for the Higgs mass. We consider all possible new colour-neutral vector-like fermions that, after electroweak symmetry breaking, mix with the SM neutrinos or charged leptons, and hence contribute to precision observables. These new leptons are predicted in many theories beyond the SM, including Grand Unified Theories (GUT) [12], models in extra dimensions [13] and Little Higgs models [14]. As they are relatively heavy, an effective Lagrangian approach should be a good approximation. In fact, we will integrate out the new leptons keeping only the operators up to dimension 6. The use of an effective formalism to fit EWPD also allows for a common treatment for any kind of new physics [15]. We leave a more general analysis for future work [16].

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<sup>1</sup>There is also a large discrepancy in the anomalous magnetic moment of the muon  $g_\mu - 2$  [9], but we do not include this observable in our fit because the contributions of the extra leptons to it are smaller than the experimental and theoretical errors. Nevertheless, at the end of Section 4 we comment on some subtle implications of  $g_\mu - 2$  through the value of the parameter  $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ .

We find that the quality of the global fit (including high- and low- $Q^2$  data) hardly improves when the new leptons are included. The case of neutrino singlets has the interesting feature of raising the preferred Higgs mass to a comfortable region above the direct LEP limit. Due to the values of  $M_W$  measured at LEP 2 and Tevatron, however, the Higgs cannot be very heavy. For other kinds of new leptons, the SM prediction for  $M_H$  is mostly unchanged.

From the global fits, we extract limits on the mixings of the different possible new leptons with the SM ones. (The limits that had been obtained before for some of these heavy leptons in [17, 18, 19] are improved.) The upper bounds on the allowed mixings range from 0.01 to 0.08 at 90 % C.L., depending on the quantum numbers of the new lepton and the family of the SM lepton it mixes with. If the new leptons are weakly coupled, the largest allowed mixings require that their masses be not far from the TeV scale.

It is important to note that new leptons with significant mixings are generically ruled out when they mediate Flavour Changing Neutral Currents (FCNC) [20, 21, 22], generate masses for the SM neutrinos [23] or contribute to neutrinoless double  $\beta$  decay [24]. To avoid these constraints, we must assume that each new lepton mixes mostly with just one family, and that their contributions to the light Majorana masses and neutrinoless double  $\beta$  decay, when allowed, are very suppressed [25]. This scenario with new Majorana particles at the TeV scale that have sizeable mixings with the SM leptons can be made natural with the help of extra symmetries. In general, these include lepton number (LN) conservation [26] and must be very slightly broken, if at all. At any rate, we find that new leptons with the quantum numbers of see-saw messengers of type I [27] and III [28] and sizable mixings can be consistent with EWPD. The neutrino singlets are also relevant to models of resonant leptogenesis [29]. All our limits apply independently of the Majorana or Dirac character of the heavy leptons, but in the Majorana case the restrictions mentioned above must be taken into account.

Finally, let us emphasize that our results are relevant to LHC, since the production and decay of these new fermions are constrained by the limits on their mixings that we give here. For production this is decisive for neutrino singlets, as they can only be produced through mixing [30]. All the other extra leptons can, in addition, be pair produced. Even if their decays are proportional to the mixings, there is enough room for the new leptons to decay within the detector [31].

The paper is organized as follows. In the next section, after a quick review

of the motivations to consider vector-like leptons, we enumerate the different possibilities and write down their couplings to the SM fields. In Section 3 we derive the effective Lagrangian describing the effect of the new leptons below threshold. We also describe the constraints from FCNC and neutrino masses. In Section 4 we introduce the observables entering the fit, and present our results for the different cases. Limits on the mixings are given in the general case and with the assumption of universality. Section 5 is devoted to a detailed discussion of the interplay between heavy lepton singlets and the Higgs mass. Section 6 contains our conclusions, including the implications of our fits for the observation of heavy leptons at large colliders. Finally, the appendix contains two tables with the experimental and SM values of the observables that we use, together with the predictions for two relevant types of new leptons.

## 2 Extending the Standard Model with vector-like leptons

Many models of physics beyond the SM include new leptons. Usually, they are vector-like, i.e. both chiralities transform in the same way under the SM gauge group. This serves to avoid constraints from gauge anomalies and also to allow masses above the electroweak scale without spoiling perturbativity. By vector-like, we refer also to Majorana fermions, for which both chiralities are not independent but related by charge conjugation. The classical example is SO(10) GUTs, which necessarily contain new singlets (the right-handed neutrinos). More recent examples include models in extra dimensions with leptons propagating in the bulk [32] and most Little Higgs models [33]. On the other hand, new leptons with masses of the order of 1 TeV and relatively large mixing with the SM leptons may be observable at future  $e^+ e^-$  colliders [34] and even at LHC in some favourable scenarios [30]. They can also give deviations in neutrino couplings, which could be measured at future neutrino experiments (see for instance [35]). Finally, these fields can induce lepton FCNC, and in some cases give mass to the light neutrinos. The current limits on the former, and the smallness of the latter impose stringent constraints, which we discuss in the next section.

It is therefore interesting to study the impact of new vector-like leptons at the TeV scale on low-energy observables, and the limits that can be derived on

their couplings and masses. To give sizable contributions to EWPD, the new leptons must mix at tree level with the SM charged leptons and/or neutrinos. This condition and the fact that the theory must be invariant under the SM gauge group restrict the quantum numbers of the new particles. All the possibilities are displayed in Table 1, which also settles our notation for the extra multiplets. We consider a generic renormalizable extension of the SM

Leptons	$N$	$E$	$\begin{pmatrix} N \\ E^- \end{pmatrix}$	$\begin{pmatrix} E^- \\ E^{--} \end{pmatrix}$	$\begin{pmatrix} E^+ \\ N \\ E^- \end{pmatrix}$	$\begin{pmatrix} N \\ E^- \\ E^{--} \end{pmatrix}$
Notation			$\Delta_1$	$\Delta_3$	$\Sigma_0$	$\Sigma_1$
$SU(2)_L \otimes U(1)_Y$	$1_0$	$1_{-1}$	$2_{-\frac{1}{2}}$	$2_{-\frac{3}{2}}$	$3_0$	$3_{-1}$
Spinor	Dirac or Majorana	Dirac	Dirac	Dirac	Dirac or Majorana	Dirac

Table 1: Lepton multiplets mixing with the SM leptons through Yukawa couplings to the SM Higgs. The electric charge is given by  $Q = T_3 + Y$ .

including these fields. After diagonalizing the kinetic and mass matrices of all the leptons in the theory (before electroweak symmetry breaking), the Lagrangian of this theory can be split into three pieces:

$$\mathcal{L} = \mathcal{L}_\ell + \mathcal{L}_h + \mathcal{L}_{\ell h}. \quad (1)$$

$\mathcal{L}_\ell$  is the SM Lagrangian and contains only light fields (with no right-handed neutrinos). We choose a basis in which the leptonic Yukawa terms are diagonal. Then the leptonic sector is given by

$$\mathcal{L}_\ell \supset \overline{l_L^i} i \not{D} l_L^i + \overline{e_R^i} i \not{D} e_R^i - \left( (\lambda_e)_i \overline{l_L^i} \phi e_R^i + \text{h.c.} \right). \quad (2)$$

Here,  $l_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$  denotes the left-handed SM doublets,  $e_R^i$  denotes the right-handed singlets,  $\phi$  is the scalar doublet  $\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ , and we use lower-case latin letters  $i, j$  as family indices.

$\mathcal{L}_h$  contains the terms involving heavy vector-like leptons and no SM leptons:

$$\mathcal{L}_h = \eta_L \overline{L^I} i \not{D} L^I - \eta_L M_I \overline{L^I} L^I - \left( (\lambda_{LL'})_{IJ} \overline{L_L^I} \Phi_{LL'} L_R^{J'} + \text{h.c.} \right). \quad (3)$$

$\overline{[L_1]} [L_2]$	$\overline{2_{-\frac{1}{2}}} 1_0$	$\overline{2_{-\frac{1}{2}}} 1_{-1}$	$\overline{2_{-\frac{3}{2}}} 1_{-1}$	$\overline{3_0} 2_{-\frac{1}{2}}$	$\overline{3_{-1}} 2_{-\frac{1}{2}}$	$\overline{3_{-1}} 2_{-\frac{3}{2}}$
$\Phi_{L_1 L_2}$	$\tilde{\phi}$	$\phi$	$\tilde{\phi}$	$\tilde{\phi}^\dagger \frac{\sigma_a}{2}$	$\phi^\dagger \frac{\sigma_a}{2}$	$\tilde{\phi}^\dagger \frac{\sigma_a}{2}$

Table 2: Form of the scalar doublet required to make the operators  $\overline{L_L} \Phi L'_R$ ,  $\overline{L_R} \Phi l_L$  and  $\overline{L_L} \Phi e_R$  gauge invariant, in terms of the quantum numbers of the leptons appearing in the operator. As usual,  $\tilde{\phi} = i\sigma_2 \phi^*$  denotes the  $Y = -1/2$  doublet.

$L_{L,R}$  stands for the two chiral components of any of the multiplets in Table 1 while  $L$  is the corresponding Dirac spinor. In the basis we are using, the mass matrices  $M$  are diagonal and real. We also allow for the possibility that  $L$  be Majorana when  $L = N$  or  $L = \Sigma_0$ , and adjust the normalization constants  $\eta_L$  with the standard values 1 and  $\frac{1}{2}$  for Dirac and Majorana spinors, respectively. The capital latin superindices  $I, J$  refer to the different exotic species with the same quantum numbers. Finally,  $\Phi_{LL'}$  represents the form of the SM scalar doublet needed for gauge invariance of the Yukawa terms, which can be read from Table 2.

The last piece,  $\mathcal{L}_{\ell h}$ , contains all the couplings between light and heavy fermions, which are of the Yukawa type:

$$\mathcal{L}_{\ell h} = -(\lambda_{Le})_{Ij} \overline{L_L^I} \Phi_{Le} e_R^j - (\lambda_{Ll})_{Ij} \overline{L_L^I} \Phi_{Ll} l_L^j + \text{h.c.} \quad (4)$$

After electroweak symmetry breaking with  $\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$ ,  $v = 246$  GeV,

mass terms mixing SM and extra leptons appear. If each SM flavour mixes at most with one extra lepton, as we shall eventually assume, the diagonalizing  $2 \times 2$  matrices are given by a mixing  $s = \sin \theta$ , up to phases. We take this mixing to be non-negative, except for  $\Sigma_1$ , where we keep a convenient relative minus sign between the mixing of  $\nu_L$  and  $e_L$ . At first order, the mixings are given by ratios of Yukawas  $\lambda$  to heavy masses  $M$  (times  $v$ ). The precise expressions for the different possible extra leptons are collected in Table 3. After the diagonalization, the charged and neutral currents for light and heavy mass eigenstates are written as a function of the lepton mixings  $s$ . The strength of the interactions involving only light leptons are modified with respect to the SM ones, correcting EWPD. This is the subject of this paper. On the other hand, the very same mixings appear in the charged and neutral currents with one light and one heavy lepton, which are relevant

	$N$	$E$	$\Delta_1$	$\Delta_3$	$\Sigma_0$	$\Sigma_1$
$s_L^\nu$	$\left  \frac{\lambda_{N1} v}{\sqrt{2} M_N} \right $	—	—	—	$\left  \frac{\lambda_{\Sigma_0} l v}{2\sqrt{2} M_{\Sigma_0}} \right $	$-\sqrt{2} s_L^e$
$s_L^e$	—	$\left  \frac{\lambda_{E1} v}{\sqrt{2} M_E} \right $	negligible	negligible	$\sqrt{2} s_L^\nu$	$\left  \frac{\lambda_{\Sigma_1} l v}{2\sqrt{2} M_{\Sigma_1}} \right $
$s_R^e$	—	negligible	$\left  \frac{\lambda_{\Delta_1 e} v}{\sqrt{2} M_{\Delta_1}} \right $	$\left  \frac{\lambda_{\Delta_3 e} v}{\sqrt{2} M_{\Delta_3}} \right $	negligible	negligible

Table 3: First order expressions in  $\frac{\lambda v}{M}$  of the mixing between one SM lepton of a given flavour and one extra lepton. Family indices are implicit and “negligible” stands for higher order contributions.

	$N$	$E$	$\Delta_1$	$\Delta_3$	$\Sigma_0$	$\Sigma_1$
$\overline{f_A} \gamma^\mu f'_A$ $\left  V_A^{ff'} \right $	$\overline{e_L^-} \gamma^\mu N_L$ $s_L^\nu$	$\overline{E_L^-} \gamma^\mu \nu_L$ $s_L^e$	$\overline{e_R^-} \gamma^\mu N_R$ $s_R^e$	$\overline{E_R^-} \gamma^\mu e_R^-$ $s_R^e$	$\overline{e_L^-} \gamma^\mu N_L$ $s_L^\nu$	$\overline{E_L^-} \gamma^\mu \nu_L$ $s_L^e$

Table 4: Resulting first order expressions of a complete subset of independent charged current couplings  $-\frac{g}{\sqrt{2}} V_A^{ff'} W_\mu^- \overline{f_A} \gamma^\mu f'_A$ ,  $A = L, R$ , as a functions of the lepton mixings.

for the production and decay of these heavy particles at large colliders. We present our results in terms of the complete subset of independent charged current couplings with one light and one heavy lepton given in Table 4. As shown in this table, they turn out to be directly related to the lepton mixings. For this reason we shall generically use the term “mixing” for both  $V$  and  $s$ .

### 3 Effective Lagrangian

As we are interested in the effects of the heavy particles at energies much smaller than their masses, we can integrate them out and use the resulting effective Lagrangian. This is completely equivalent, for our purposes, to diagonalizing the mass matrices to first order and using the resulting charged and neutral couplings for light fields. Nevertheless, we find it interesting to write down the completely-gauge-invariant induced operators and their coefficients before electroweak symmetry breakdown. In particular, this may

be useful to compare with other new physics effects in EWPD. Because the heavy leptons are vector-like, they decouple in the limit when their mass goes to infinity. Therefore, we expand the effective Lagrangian as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_4 + \frac{1}{\Lambda} \mathcal{L}_5 + \frac{1}{\Lambda^2} \mathcal{L}_6 + \dots \quad (5)$$

where each  $\mathcal{L}_d$  contains gauge-invariant local operators of canonical dimension  $d$ , and the scale  $\Lambda$  is equal to the mass  $M$  of the lightest new lepton. The operators in  $\mathcal{L}_d$  give contributions of order  $(E/\Lambda)^{d-4}$  to observables, with  $E$  the typical energy of the processes involved or the vacuum expectation value  $v$  of the scalar field. We expect the terms of dimension  $d > 6$  to give small corrections compared to the experimental precision of current data, so we neglect them in the fits. Our results will be consistent with this approximation.

In  $\mathcal{L}_5$  there is only one operator:

$$\mathcal{L}_5 = (\alpha_5)_{ij} \overline{(l_L^i)^c} \tilde{\phi}^* \tilde{\phi}^\dagger l_L^j + \text{h.c.} \quad (6)$$

This is the lepton number violating Weinberg operator [36], which after electroweak symmetry breaking gives masses to the light neutrinos,  $m_\nu = -v^2 \alpha_5 / \Lambda$ , with  $(\alpha_5)_{ee}$  contributing also to neutrinoless double  $\beta$  decay. This operator can originate from Majorana terms in (3), which are possible only for extra singlets or triplets of zero hypercharge. The value of the coefficient  $\alpha_5$  is given in Table 5. The fact that neutrino masses are tiny, and the strict bounds on neutrino double  $\beta$  decay, are usually explained by a large scale  $\Lambda$ . However, we want to keep the scale  $\Lambda$  near the TeV range to have non-negligible effects from  $\mathcal{L}_6$ . Then we need to assume that some mechanism in the high energy model keeps the coefficient  $(\alpha_5)_{ij}$  very small. A natural way to achieve this in any model is to implement lepton number conservation, up to possible breaking terms with adimensional coefficients  $\alpha_5$  smaller than  $10^{-11}$  [23]. This scenario is stable under quantum corrections and is realized in models in which the heavy fermions are of Dirac type [26]. Unnatural cancellations are also possible [25].

At order  $1/\Lambda^2$ , we find

$$\begin{aligned} \mathcal{L}_6 = & \left( \alpha_{\phi l}^{(1)} \right)_{ij} (\phi^\dagger i D_\mu \phi) \left( \overline{l_L^i} \gamma^\mu l_L^j \right) + \left( \alpha_{\phi l}^{(3)} \right)_{ij} (\phi^\dagger i \sigma_a D_\mu \phi) \left( \overline{l_L^i} \sigma_a \gamma^\mu l_L^j \right) + \\ & + \left( \alpha_{\phi e}^{(1)} \right)_{ij} (\phi^\dagger i D_\mu \phi) \left( \overline{e_R^i} \gamma^\mu e_R^j \right) + (\alpha_{e\phi})_{ij} (\phi^\dagger \phi) \overline{l_L^i} \phi e_R^j + \text{h.c..} \end{aligned} \quad (7)$$

We have made field redefinitions to write the operators in the basis of Buchmuller and Wyler [37], and follow the notation in this reference. The values of the coefficients of the operators are given in Tables 5 and 6. These results parallel the ones obtained and discussed in [38] for extra quarks. After electroweak symmetry breaking these operators modify the neutral current and charged current couplings of leptons:

$$\begin{aligned}\delta g_L^\nu &= \frac{1}{4} \left( -\alpha_{\phi l}^{(1)} + \alpha_{\phi l}^{(3)} + \text{h.c.} \right) \frac{v^2}{\Lambda^2} \\ \delta g_L^e &= -\frac{1}{4} \left( \alpha_{\phi l}^{(1)} + \alpha_{\phi l}^{(3)} + \text{h.c.} \right) \frac{v^2}{\Lambda^2} \\ \delta g_R^e &= -\frac{1}{4} \left( \alpha_{\phi e}^{(1)} + \text{h.c.} \right) \frac{v^2}{\Lambda^2} \\ \delta V_L^{e\nu} &= \left( \alpha_{\phi l}^{(3)} \right)^\dagger \frac{v^2}{\Lambda^2}.\end{aligned}\tag{8}$$

Here, the  $\delta g$  and  $\delta V$  are in principle general matrices. The charged lepton masses and their Yukawa couplings to the Higgs are also modified, but these changes can be absorbed into the observed charged lepton masses. Moreover, neglecting tiny irrelevant contributions from neutrino masses, we can re-diagonalize the mass matrix with bi-unitary transformations that do not introduce further changes in the neutral and charged currents to order  $1/\Lambda^2$ . So, the operator  $(\phi^\dagger \phi) \bar{l}_L^j \phi e_R^j$  and the corresponding coefficients  $\alpha_{e\phi}$  in Tables 5 and 6 do not contribute to our fits. Observe also in Table 5 that the combinations of Yukawa couplings entering  $\alpha_{\phi l,e}^{(1,3)}$  is different from the ones in  $\alpha_5$ , so that it is perfectly possible to have finite  $\alpha_{\phi l,e}^{(1,3)}$  and vanishing  $\alpha_5$  simultaneously, even for  $N$  and  $\Sigma_0$  multiplets [23].

On the other hand, the off-diagonal elements of the coefficient matrices  $\alpha_{\phi l,e}^{(1,3)}$  induce leptonic FCNC. The current experimental limits on rare processes like  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow eee$  imply that these off-diagonal coefficients are small [21]. As can be seen from Table 5, this requires that each new fermion multiplet mixes mostly with only one of the known lepton flavours. This pattern of mixings is automatic with the extra assumption of an (approximate) conservation of individual lepton number.

$L$	$\frac{\alpha_5}{\Lambda}$	$\frac{\alpha_{\phi l}^{(1)}}{\Lambda^2}$	$\frac{\alpha_{\phi l}^{(3)}}{\Lambda^2}$	$\frac{\alpha_{\phi e}^{(1)}}{\Lambda^2}$	$\frac{\alpha_{e\phi}}{\Lambda^2}$
$N$	$\frac{1}{2}\lambda_{Nl}^T M_N^{-1} \lambda_{Nl}$	$\frac{1}{4}\lambda_{Nl}^\dagger M_N^{-2} \lambda_{Nl}$	$-\frac{\alpha_{\phi l}^{(1)}}{\Lambda^2}$	—	—
$E$	—	$-\frac{1}{4}\lambda_{El}^\dagger M_E^{-2} \lambda_{El}$	$\frac{\alpha_{\phi l}^{(1)}}{\Lambda^2}$	—	$-2\frac{\alpha_{\phi l}^{(1)}}{\Lambda^2} \lambda_e$
$\Delta_1$	—	—	—	$\frac{1}{2}\lambda_{\Delta_1 e}^\dagger M_{\Delta_1}^{-2} \lambda_{\Delta_1 e}$	$\lambda_e \frac{\alpha_{\phi e}^{(1)}}{\Lambda^2}$
$\Delta_3$	—	—	—	$-\frac{1}{2}\lambda_{\Delta_3 e}^\dagger M_{\Delta_3}^{-2} \lambda_{\Delta_3 e}$	$-\lambda_e \frac{\alpha_{\phi e}^{(1)}}{\Lambda^2}$
$\Sigma_0$	$\frac{1}{8}\lambda_{\Sigma_0 l}^T M_{\Sigma_0}^{-1} \lambda_{\Sigma_0 l}$	$\frac{3}{16}\lambda_{\Sigma_0 l}^\dagger M_{\Sigma_0}^{-2} \lambda_{\Sigma_0 l}$	$\frac{1}{3}\frac{\alpha_{\phi l}^{(1)}}{\Lambda^2}$	—	$\frac{4}{3}\frac{\alpha_{\phi l}^{(1)}}{\Lambda^2} \lambda_e$
$\Sigma_1$	—	$-\frac{3}{16}\lambda_{\Sigma_1 l}^\dagger M_{\Sigma_1}^{-2} \lambda_{\Sigma_1 l}$	$-\frac{1}{3}\frac{\alpha_{\phi l}^{(1)}}{\Lambda^2}$	—	$-\frac{2}{3}\frac{\alpha_{\phi l}^{(1)}}{\Lambda^2} \lambda_e$

Table 5: Coefficients of the operators arising from the integration of heavy leptons. The dimension five operator entry,  $\frac{\alpha_5}{\Lambda}$ , only appears when the singlet  $N$  and/or the triplet  $\Sigma_0$  are Majorana fermions.

## 4 Global fit

We have performed global fits to the existing EWPD to confront the hypothesis of new leptons with the SM, and to constrain the new parameters (lepton mixings). In the appendix, we show in Tables 11 and 12 the observables that enter our fits, together with their current experimental values and the SM predictions. We do not include data at higher energies from LEP 2 because they do not change significantly the fits. The reason is that the Z-pole observables have better precision and constrain strongly all the new parameters in the model, i.e. no independent parameters enter the LEP 2 data. This can be understood by the fact that the new leptons change only the trilinear couplings, and do not generate four-fermion operators in the effective Lagrangian.

With the experimental data we construct the  $\chi^2$  function to be minimized:

$$\chi^2(\theta) = [\mathbf{O}_{\text{exp}} - \mathbf{O}_{\text{th}}(\theta)]^T U_{\text{exp}}^{-1} [\mathbf{O}_{\text{exp}} - \mathbf{O}_{\text{th}}(\theta)], \quad (9)$$

where  $(U_{\text{exp}})_{ij} = \sigma_i \rho_{ij} \sigma_j$  is the covariance matrix, with  $\sigma$  the experimental errors and  $\rho$  the correlation matrix, and  $\theta$  are the free parameters. In  $U_{\text{exp}}$  we include both statistical and systematic errors.  $\mathbf{O}_{\text{exp}}$  are the experimental values of the (pseudo) observables and  $\mathbf{O}_{\text{th}}(\theta)$  contains the theoretical predictions obtained from  $\mathcal{L}_{\text{eff}}$  and expressed in terms of the parameters of the original model (SM + new leptons). The good agreement of the SM

$L_1, L_2$	$\frac{\alpha_{e\phi}}{\Lambda^2}$
$E, \Delta_1$	$\lambda_{El}^\dagger M_E^{-1} \lambda_{E\Delta_1} M_{\Delta_1}^{-1} \lambda_{\Delta_1 e}$
$E, \Delta_3$	$\lambda_{El}^\dagger M_E^{-1} \lambda_{E\Delta_3} M_{\Delta_3}^{-1} \lambda_{\Delta_3 e}$
$\Delta_1, \Sigma_0$	$\frac{1}{2} \lambda_{\Sigma_0 l}^\dagger M_{\Sigma_0}^{-1} \lambda_{\Sigma_0 \Delta_1} M_{\Delta_1}^{-1} \lambda_{\Delta_1 e}$
$\Delta_1, \Sigma_1$	$\frac{1}{4} \lambda_{\Sigma_1 l}^\dagger M_{\Sigma_1}^{-1} \lambda_{\Sigma_1 \Delta_1} M_{\Delta_1}^{-1} \lambda_{\Delta_1 e}$
$\Delta_3, \Sigma_1$	$-\frac{1}{4} \lambda_{\Sigma_1 l}^\dagger M_{\Sigma_1}^{-1} \lambda_{\Sigma_1 \Delta_3} M_{\Delta_3}^{-1} \lambda_{\Delta_3 e}$

Table 6: Combined contribution to  $\alpha_{e\phi}$  from the simultaneous integration of different mixed multiplets. Even if the corresponding operator does not affect our fits, we include the values of the coefficient for completeness.

with the experimental data allows us to consider only the corrections coming from the interference between the SM and the new pieces in the effective Lagrangian. This means that we calculate only tree-level contributions from new physics, and linearize the values of the observables in  $v^2/\Lambda^2$ . We use ZFITTER 6.42 [39] to compute the SM predictions at the quantum level.

Within our approximations, the new free parameters of the model always enter the fit as ratios of Yukawa couplings to heavy masses, corresponding to the mixing between light and heavy particles as explained above and gathered in Table 3. We present our results in function of the equivalent charged current couplings  $V$  in Table 4. The fits constrain only the magnitudes  $|V|$ . The new leptons can modify the observables in two ways. First, they can give direct contributions to the processes relevant to a given observable. Second, they can contribute to the processes from which the input parameters are extracted. This changes the relation between the measured values and the SM parameters, and results in indirect corrections to all the observables.

The free parameters in the fits are  $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ ,  $\alpha_S(M_Z^2)$ ,  $M_Z$ ,  $m_t$ ,  $M_H$  and the mixings of the new leptons. Note that the first four parameters are to a great extent determined by the corresponding experimental measurements<sup>2</sup>. Therefore, only  $M_H$  and the mixings can vary significantly and we will give the results in terms of these two parameters. Furthermore, we make use of

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<sup>2</sup>We can neglect the effect of the heavy leptons on these measurements. In particular, for  $\alpha_S$  we take the world average in [1]. Even if this average includes the SM fit to EWPD as an input, the central value will not be changed significantly by the presence of new leptons, and we use the most conservative error interval given in that reference.

the information from direct Higgs searches at LEP by imposing a sharp lower cut-off on the Higgs mass,  $M_H \geq 114.4$  GeV. This is a good approximation to the more precise treatment proposed in [40].

The minimization of  $\chi^2$  and the calculation of the confidence regions are performed by scanning over the parameter space<sup>3</sup> and accepting or rejecting points according to their probability. The plots are obtained from the actual sets of points, keeping only the points within the 90% probability regions and performing a coarse graining to lower the size of the figures.

## 4.1 Numerical results

Coupling	$n_{par}^{new}$	$N$	$E$	$-\Delta\chi^2_{min}$	$(\chi^2_{min}/\text{d.o.f.})$		
				$\Delta_1$	$\Delta_3$	$\Sigma_0$	$\Sigma_1$
General	3	1.5 (1.57)	0.5 (1.61)	1.9 (1.56)	1.5 (1.57)	1.3 (1.58)	0 (1.63)
Universal	1	1.0 (1.44)	0 (1.49)	0 (1.49)	0.3 (1.48)	0.7 (1.46)	0 (1.49)
Only with $e$	1	0.8 (1.49)	0 (1.51)	0 (1.51)	0.7 (1.49)	1.0 (1.48)	0 (1.51)
Only with $\mu$	1	1.0 (1.48)	0.5 (1.50)	1.9 (1.45)	0 (1.51)	0 (1.51)	0 (1.51)
Only with $\tau$	1	1.0 (1.48)	0 (1.51)	0 (1.51)	0.6 (1.49)	0.2 (1.51)	0 (1.51)

Table 7: Decrease in  $\chi^2_{min}$  with respect to the SM minimum,  $\chi^2_{SM} = 43.92$  ( $\chi^2_{SM} = 29.82$  with lepton universality), obtained by adding to the SM the different leptons. The number of degrees of freedom is obtained as  $\mathcal{N} - 5 - n_{par}^{new}$ , where  $n_{par}^{new}$  is the number of independent lepton mixings and  $\mathcal{N} = 35$  is the number of observables ( $\mathcal{N} = 26$  for the universal case). In parenthesis we write the value of  $\chi^2_{min}/\text{d.o.f.}$ , which for the SM is 1.46 (1.42 with lepton universality).

In Table 7 we show the improvements  $-\Delta\chi^2_{min}$  with respect to the SM minimum (consistent with  $M_H \geq 114.4$  GeV), when we add independently one kind of new lepton at a time. We have also performed a general fit including all possible heavy leptons, but there is no further significant improvement and we do not show the result here. We distinguish different scenarios depending on how we choose the couplings of the new leptons to the SM fields. We have considered the following cases:

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<sup>3</sup>In practice, for the reasons discussed above, we restrict the parameters  $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ ,  $\alpha_S(M_Z^2)$ ,  $M_Z$ ,  $m_t$  to  $1\sigma$  intervals around their SM value.

- A single new lepton coupled only to one of the three SM families (“Only with  $e$ ,  $\mu$  or  $\tau$ ”).
- Three leptons, each coupled to one (different) SM family with independent couplings (“General”).
- Three leptons, each coupled to one (different) SM family with the same coupling (“Universal”).

The universal case requires an extra assumption. When we do the fit with universal couplings, we use this assumption of universality also in the experimental measurements. This implies that the set of data is different and hence the comparison with the other fits is not direct. The (pseudo) observables included in the fit for this case, with their current experimental values and the SM predictions, are collected in Table 12 in the appendix.

We see that there are mild improvements with respect to the SM  $\chi^2$  for singlets  $N_{\mu,\tau}$ , doublets  $(\Delta_1)_\mu$  and triplets  $(\Sigma_0)_e$ , and also for universal singlets  $N$ . In all the other cases the  $\chi^2$  is lowered by less than one unit. The only fit with  $\chi^2/\text{d.o.f.}$  smaller than in the SM is obtained for the SM-like doublet coupled to the second family,  $(\Delta_1)_\mu$ . Even if the improvements are marginal at best, it is interesting that in some cases the minima occur for significant values of the mixings, as can be seen in Table 8. Let us also mention the biggest changes in individual observables at the global minima. First,  $\sigma_H^0$  (with a 1.7 pull in the SM) is improved in several cases, up to a pull of 0.8 for the singlet  $N_\tau$ . The pull in the SLD asymmetry  $A_e$  is lowered from 2.0 to 1.7 for singlets  $N_{e,\mu}$ , but at the price of increasing the  $A_{FB}^{0,b}$  anomaly from 2.6 to 2.8. The NuTeV anomaly is reduced only for universal triplets  $\Sigma_0$ , and only from 2.8 to 2.6. Finally,  $(\Delta_1)_\mu$ , reduces the pull in  $R_\mu^0$  from 1.4 to 0.1. In Tables 11 and 12 we give, together with the experimental and SM values, the best-fit values for our set of observables in the extensions with a doublet  $(\Delta_1)_\mu$  and with a universal singlet  $N$ , respectively.

From the fits, we can also extract limits on the values of the mixings  $V$  and  $s$  in Tables 3 and 4, respectively. We give the 90% C.L. upper bounds on the absolute value of  $V$  in Table 8. We stress again that these limits incorporate the information from the direct Higgs searches.

In Figs 1 to 6 we show the 90% C.L. regions in the  $|V| - M_H$  parameter space. In these plots we display in blue the 90% probability region of the fit without any restriction on  $M_H$ , and in black the extension of the 90%

Coupling		$N$	$E$	$\Delta_1$	$\Delta_3$	$\Sigma_0$	$\Sigma_1$
Only with $e$	$ V  <$	0.055	0.018	0.018	0.025	0.019	0.013
	$ V_{\min}  =$	0.035	0	0	0.018	0.014	0
Only with $\mu$	$ V  <$	0.057	0.034	0.045	0.024	0.017	0.022
	$ V_{\min}  =$	0.036	0.020	0.035	0	0	0
Only with $\tau$	$ V  <$	0.079	0.030	0.030	0.042	0.027	0.026
	$ V_{\min}  =$	0.057	0	0	0.028	0.015	0
Universal	$ V  <$	0.038	0.018	0.019	0.022	0.016	0.011
	$ V_{\min}  =$	0.025	0	0	0.014	0.012	0

Table 8: Upper limit at 90 % C.L. on the absolute value of the mixings in Table 4 and their value at the minimum. The first three rows are obtained by coupling each new lepton with only one SM family. The last one corresponds to the case of lepton universality. All numbers are computed assuming  $M_H \geq 114.4$  GeV.

region when we enforce  $M_H \geq 114.4$  GeV. The direct lower limit on  $M_H$  is represented by the vertical line.

As is apparent in the plots, in some cases there is a correlation between the mixing and  $M_H$ . In particular, we can see in Fig. 1 a strong positive correlation for the singlet  $N$ , as long as it mixes with the first and/or second family of SM leptons. As a result, the preferred Higgs mass is larger than in the SM<sup>4</sup>. This is in fact responsible for part of the improvement in the  $\chi^2$  in this case. We analyze the interplay between the Higgs mass and the mixing of neutrino singlets in more detail in the next section. In Table 9 we give the 90% C.L. upper limits that we find in the different scenarios. These limits take into account the direct lower bound. The limits with extra singlets are significantly weaker than in the SM.

Because  $A_{FB}^{0,b}$  and  $g_L^2$  show discrepancies beyond  $2.6\sigma$ , in the SM and in all the extensions with leptons—except for  $(\Delta_3)_e$ , which gives a slightly smaller pull of 2.4 for  $A_{FB}^{0,b}$ —it is reasonable to consider them as outliers

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<sup>4</sup>This effect has been discussed before by Loinaz et al. in [11]. In that reference, a much heavier Higgs is allowed because the constraint from  $M_W$  is not enforced (or it is compensated by unknown new physics). We discuss the differences between our analysis and the one in [11] below.

Couplings		$N$	$E$	$\Delta_1$	$\Delta_3$	$\Sigma_0$	$\Sigma_1$
Only with $e$	$M_H$ [GeV] <	259	166	168	162	168	163
Only with $\mu$	$M_H$ [GeV] <	267	187	167	165	163	162
Only with $\tau$	$M_H$ [GeV] <	164	165	167	164	167	166
Universal	$M_H$ [GeV] <	253	171	170	163	166	164

Table 9: Upper limit at 90 % C.L. on the Higgs mass (in GeV). The first three rows are obtained by coupling each new lepton with only one SM family. The last one correspond to the case of lepton universality. All numbers are computed assuming  $M_H \geq 114.4$  GeV.

that should be removed from the fits. This is indeed the correct approach if the anomalies are due to underestimated systematic errors or to (additional) new physics which does not modify other observables. In the SM, this would make the preferred Higgs mass much lower than the direct LEP limit [4]. To quantify to what extent this effect is problematic nowadays and see whether the situation is improved by extra leptonic singlets, we have repeated the fits for the SM and for universal neutrino singlets excluding all low energy observables and  $A_{FB}^{0,b}$ , and imposing again the constraint  $M_H \geq 114.4$  GeV. We find  $\chi^2/\text{d.o.f.} = 11.1/13$  in the SM and  $\chi^2/\text{d.o.f.} = 7.7/12$  for extra singlets. Therefore, we see that there is a significant improvement in the quality of this fit when we include new singlets. This comes in part from the fact that a bigger improvement in  $A_l(\text{SLD})$  is possible when  $A_{FB}^{0,b}$  is disregarded. On the other hand, it is also apparent that the SM is perfectly consistent with this reduced set of data, even with the constraint from the direct Higgs searches, with a probability of 60.2 % of a larger  $\chi^2$ .

In fact, in general the SM “adapts” to relatively large values of  $M_H$  by lowering and increasing a bit the values of  $\Delta\alpha_{\text{had}}^{(5)}$  and  $m_t$ , respectively. This is not necessary with extra neutrino singlets coupled to the first or second families. In this regard, let us note that  $g_\mu - 2$  prefers higher values of  $\Delta\alpha_{\text{had}}^{(5)}$ , so that including it in the fits would favour the extension with singlets with respect to the SM [41].

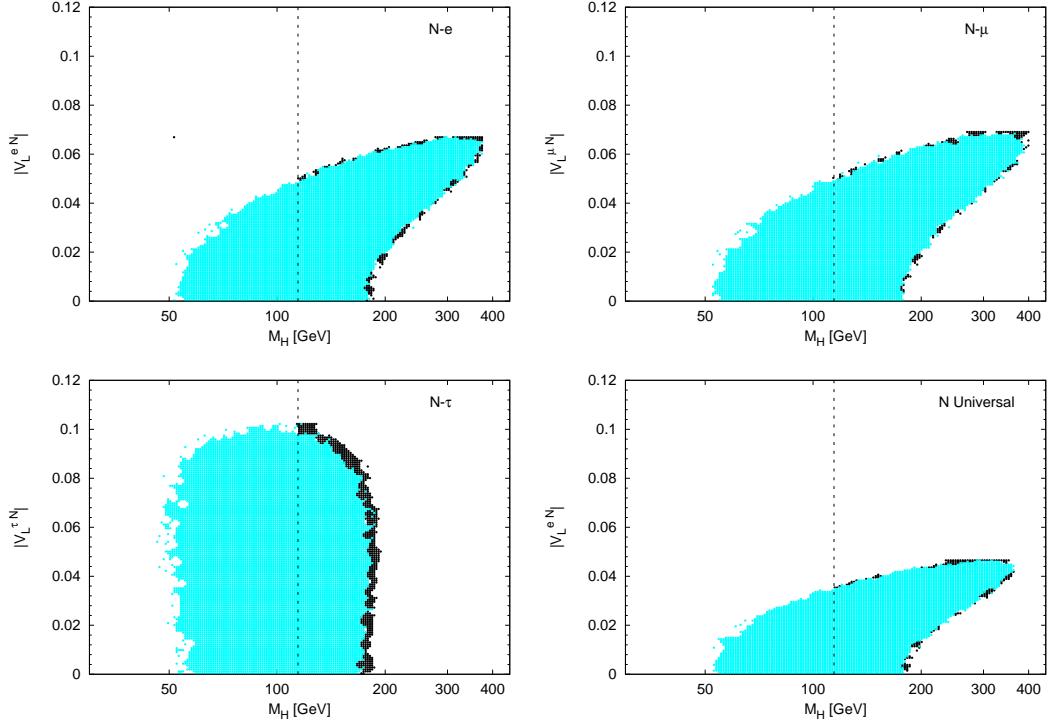


Figure 1: 90% confidence region in the  $|V_L^{eN}| - M_H$  parameter space for the  $N$  singlet coupled to the first, second and third family, respectively. The last plot corresponds to the universal case. In all cases the extension of the 90% confidence region with the cut  $M_H \geq 114.4$  GeV (represented by the vertical dashed line) is shown in black.

## 5 Large neutrino mixing and the Higgs mass

From Table 8, we see that the less constrained extra leptons are the neutrino singlets. These fields can play the role of see-saw messengers, although as we have mentioned their contribution to  $\alpha_5$  must be suppressed or cancelled by another contribution. In this section we analize this case in detail, emphasizing the role of the Higgs boson.

The mixing of new leptons with the light neutrinos modifies the invisible width of the  $Z$ ,  $\Gamma_{\text{inv}}$ . This shifts the prediction for  $\sigma_H^0$  in the opposite

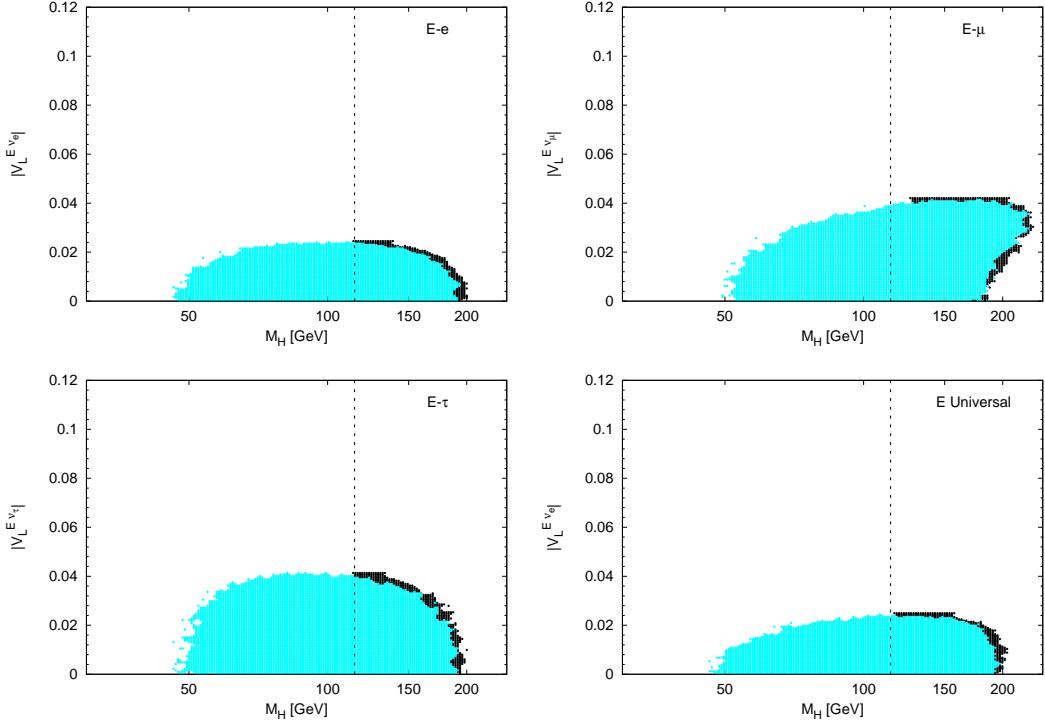


Figure 2: 90% confidence region in the  $|V_L^{E\nu}| - M_H$  parameter space for the  $E$  singlet coupled to the first, second and third family, respectively. The last plot corresponds to the universal case. In all cases, the extension of the 90% confidence region with the cut  $M_H \geq 114.4$  GeV (represented by the vertical dashed line) is shown in black.

direction, since

$$\sigma_H^0 = 12\pi \frac{\Gamma_e \Gamma_h}{M_Z^2 \Gamma_Z^2}, \quad (10)$$

and  $\Gamma_Z = \Gamma_l + \Gamma_h + \Gamma_{\text{inv}}$  (with the leptonic width  $\Gamma_l = 3\Gamma_e$  in the universal case). For the singlets  $N$ , the invisible width is smaller and the shift in  $\sigma_H^0$  is positive, so the pull in this quantity is reduced. These are the only effects on Z-pole observables when the new singlet mixes only with the third family. On the other hand, the independence of these couplings for different families is limited in the fit by the decays of the  $W^\pm$ , which do not allow for big departures from universality in the neutrino couplings. For this reason, the pull decreases only from 1.7 in the SM to 0.8.

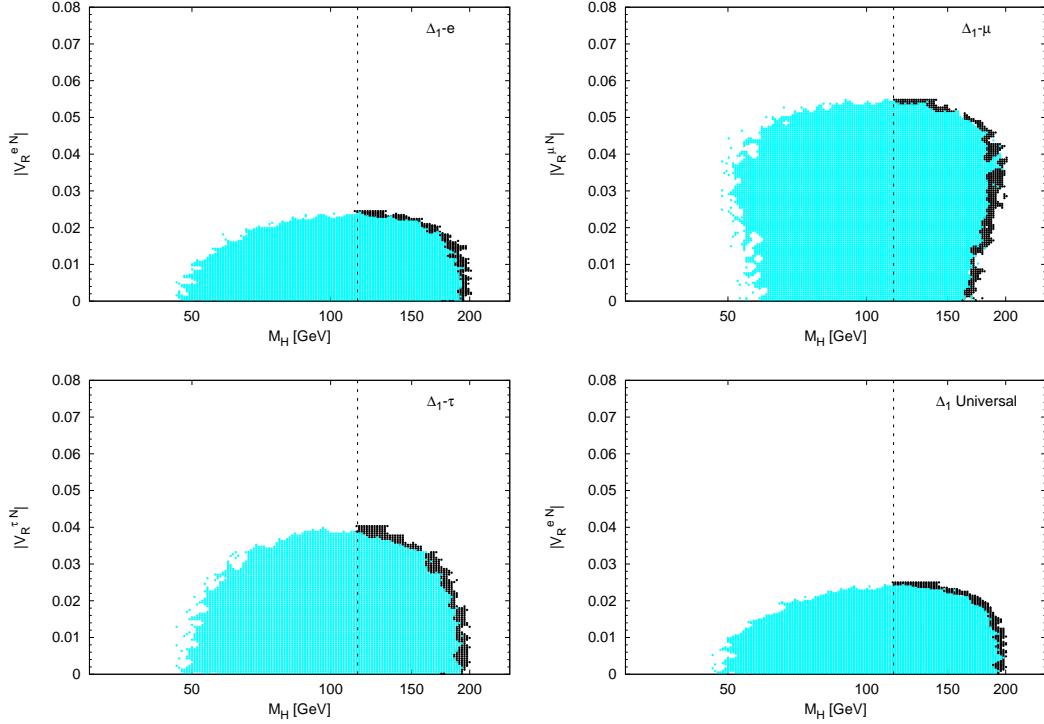


Figure 3: 90% confidence region in the  $|V_R^{eN}| - M_H$  parameter space for the  $\Delta_1$  doublet coupled to the first, second and third family, respectively. The last plot corresponds to the universal case. In all cases, the extension of the 90% confidence region with the cut  $M_H \geq 114.4$  GeV (represented by the vertical dashed line) is shown in black.

A more interesting feature appears as the result of the coupling of  $N$  to the first two families. These couplings generate the operators  $\left(\mathcal{O}_{\phi l}^{(3)}\right)_{ee,\mu\mu}$ , which contribute to muon decay and affect the extraction of the Fermi constant  $G_F$  from the muon lifetime. Because  $G_F$  is an input parameter, this effect propagates to all observables, giving indirect corrections that mimic the ones of the  $T$  oblique parameter of Peskin and Takeuchi [42]. With the normalization of [43],

$$\hat{T}_{\text{eff}} = -\text{Re} \left[ \left( \alpha_{\phi L}^{(3)} \right)_{ee} + \left( \alpha_{\phi L}^{(3)} \right)_{\mu\mu} \right] \frac{v^2}{\Lambda^2}. \quad (11)$$

This equation applies to all our observables except  $M_W$ , which is discussed

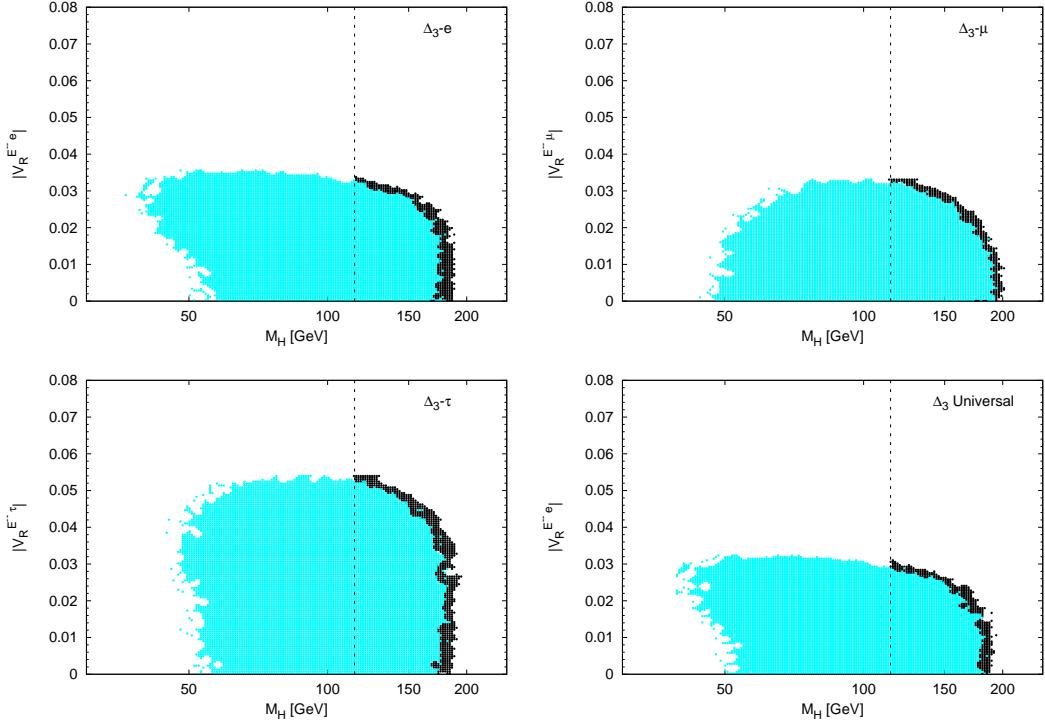


Figure 4: 90% confidence region in the  $|V_R^{E--e}| - M_H$  parameter space for the  $\Delta_3$  doublet coupled to the first, second and third family, respectively. The last plot corresponds to the universal case. In all cases, the extension the 90% confidence region with the cut  $M_H \geq 114.4$  GeV (represented by the vertical dashed line) is shown in black.

below.

As the dominant effects of the Higgs boson are oblique as well, some cancellations can take place. Indeed, including the leading contribution of the Higgs mass and the shift in  $G_F$ , the corrections to the oblique parameter  $\epsilon_1$  [44] are given by

$$\delta\epsilon_1 = -\frac{3G_FM_W^2}{4\sqrt{2}\pi^2} \tan^2\theta_W \log \frac{M_H}{M_Z} + \hat{T}_{\text{eff}}. \quad (12)$$

Hence, we see that the effect in  $\epsilon_1$  of a heavy Higgs mass can be compensated by a positive value of  $\hat{T}_{\text{eff}}$ . In fact, it is known that a heavy Higgs can be made consistent with EWPD by new oblique physics that gives a positive

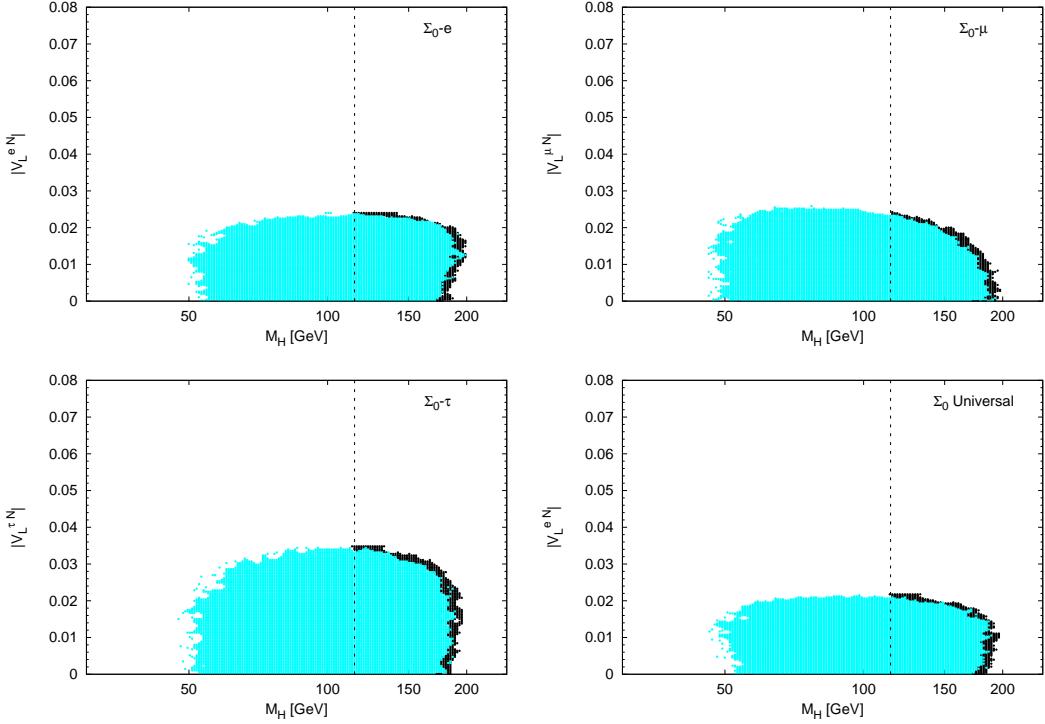


Figure 5: 90% confidence region in the  $|V_L^{eN}| - M_H$  parameter space for the  $\Sigma_0$  triplet coupled to the first, second and third family, respectively. The last plot corresponds to the universal case. In all cases the 90% confidence region with the cut  $M_H \geq 114.4$  GeV (represented by the vertical dashed line) is shown in black.

$T$  parameter, even if the positive contributions of the Higgs to  $\epsilon_3$  are not cancelled by a negative  $S$  parameter. For the neutrino singlets, the sign of  $\hat{T}_{\text{eff}}$  is actually positive. This, combined with the improvement in the hadronic width, explains that the fit prefers relatively large values of  $M_H$ , as can be seen in Tables 9 and 10. In Fig. 1 we observe clearly how a non-vanishing mixing of new singlets with electron and/or muon neutrinos allows for larger values of  $M_H$ , thus eliminating the (mild) tension between the global electroweak fit and the direct LEP lower bound on  $M_H$ .

On the other hand, unlike the shift in  $G_F$ , a genuine  $T$  parameter from new oblique physics would give additional direct contributions to  $M_W$  (for fixed  $M_Z$ ). These are not included in our  $\hat{T}_{\text{eff}}$ , and in general cannot be

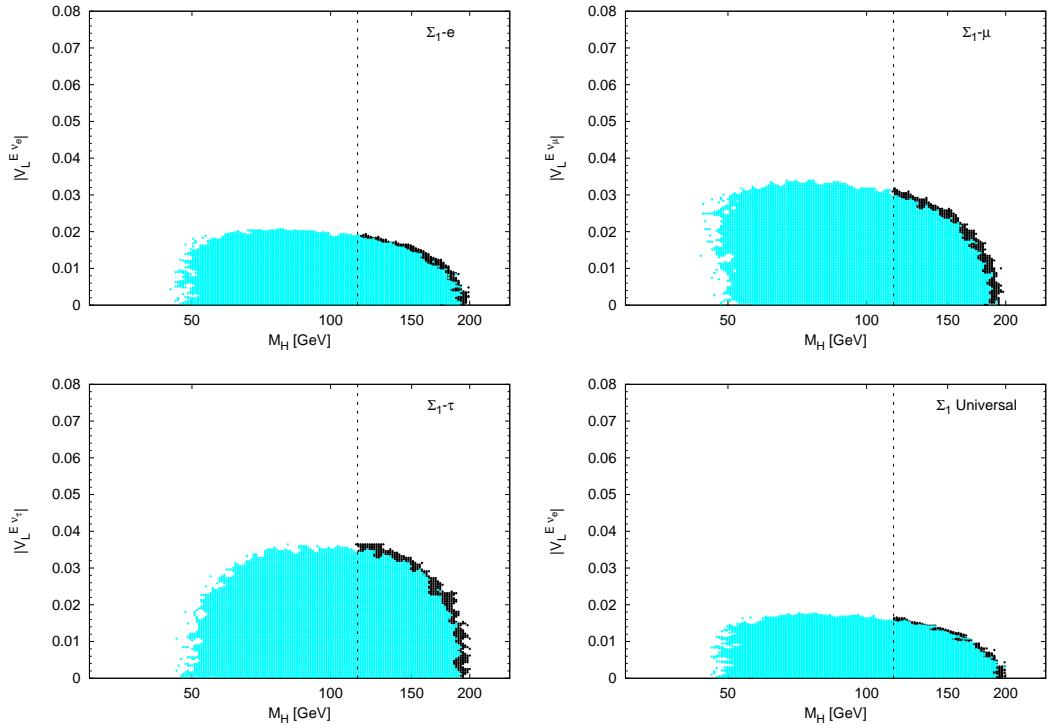


Figure 6: 90% confidence region in the  $|V_L^{E\nu}| - M_H$  parameter space for the  $\Sigma_1$  triplet coupled to the first, second and third family, respectively. The last plot corresponds to the universal case. In all cases the 90% confidence region with the cut  $M_H \geq 114.4$  GeV (represented by the vertical dashed line) is shown in black.

generated by any kind of new leptons at tree level. A heavy Higgs gives the complete  $T$ -like contributions (in addition to  $S$ -like and suppressed  $U$ -like contributions). Therefore, there is no cancellation of Higgs and singlet effects in  $M_W$ , once the relation between mixings and  $M_H$  has been determined from Z-pole observables. This prevents the Higgs from being too heavy, and the lepton mixings from being too large.

Let us also note that the net contribution of the new singlets to neutrino–nucleon deep inelastic scattering is suppressed, due to an approximate cancellation between their indirect and direct effects. Therefore, the dominant effect is the oblique Higgs boson contribution, which is negative when  $M_H$  is

$N$ Coupling	$e$	$\mu$	$\tau$	Universal
$M_H$ [GeV]	132.4	135.9	114.4	135.4

Table 10: Best-fit values of the Higgs mass (in GeV) for the extensions with neutrino singlets. In all the other cases the Higgs mass prefers to remain at the imposed cut,  $M_H = 114.4$  GeV.

increased with respect to the reference value<sup>5</sup>. This would explain the NuTeV anomaly if the Higgs were allowed to be very heavy. But as we have discussed above,  $M_W$  prefers a light Higgs, and in the best fit to all observables there is no improvement in  $g_L^2$ .

Our conclusions are not at odds with the one of Loinaz *et al.* in [11]<sup>6</sup>. They claim that mixing of light and heavy neutrinos can account for the NuTeV anomaly and, together with a heavy Higgs, give an excellent fit *as long as*  $M_W$  is not included in the fit or additional new physics supplies a big  $U$  parameter. We have preferred, instead, to include  $M_W$  in our fits, as this observable is well measured nowadays. Moreover, dimensional and symmetry arguments suggest that, in the absence of fine tuning,  $U$  is smaller than  $T$  for any new physics coming in at a scale larger than  $M_W$  [42, 43]. This is indeed found in known calculable models. So, it seems difficult that any new physics can yield the values  $U \gg T$  required in the fit of [11]<sup>7</sup>. When  $M_W$  is included in the global fit and no *ad hoc*  $U$  parameter is introduced to eliminate its influence, the results are not that spectacular. We find that the Higgs cannot be very heavy and that the NuTeV anomaly is not explained. Nevertheless, as we have discussed, there is an improvement in  $\sigma_H^0$  (through the invisible width) and a Higgs heavier than in the SM is allowed.

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<sup>5</sup>Alternatively, the Z-pole observables impose an approximate cancellation between the  $M_H$  and  $\hat{T}_{\text{eff}}$  contributions. This leaves the negative direct contribution of the new singlets.

<sup>6</sup>As a technical point, let us mention that our formulas for  $g_L^2$  and  $g_R^2$  in neutrino deep inelastic scattering differ from the ones in this reference, because we include the heavy-lepton contributions to the determination of  $V_{ud}$  from  $\beta$  decay, just as we did for  $G_F$  and muon decay. These contributions reverse the sign of the singlet contributions to  $g_L$ , which then play against the improvement of the NuTeV anomaly. However, in both cases the singlet contributions are subleading with respect to the Higgs ones and do not alter the qualitative conclusions.

<sup>7</sup>We do not claim that this possibility is logically excluded. The authors of [11] propose the possibility that threshold effects in a strongly coupled sector might give rise to the necessary enhancement of  $U$ .

## 6 Conclusions

We have performed a global fit to existing EWPD for extensions of the SM with new vector-like leptons. The analysis makes use of the corresponding effective Lagrangian up to dimension 6, which is justified by the smallness of the mixings we find. Our main results are displayed in Tables 8 and 9, and illustrated in the different plots. In the cases that had been analyzed before [17, 18, 19], we find more stringent limits (at the few per cent level). This reflects the better agreement of the SM predictions with the present data.

In Table 7, we give the improvements in the  $\chi^2$  of the global fit when the SM is supplemented by new leptons. The addition of more than one type of extra lepton multiplet at a time does not improve the quality of the fit. The  $\chi^2/\text{d.o.f.}$  is (slightly) reduced with respect to the one in the SM for  $(\Delta_1)_\mu$  only. Even if we do not find any significant improvement of the SM global fit, it is interesting to observe that TeV-scale vector-like leptons with sizeable mixings are consistent with EWPD. An interesting feature of the fits is that the presence of extra singlets mixing with the electron and/or muon neutrinos favours higher values of the Higgs mass, which lie comfortably in the region allowed by direct searches of the Higgs at LEP. This accounts for part of the improvement in the  $\chi^2$  in these cases, and implies significantly weaker upper bounds on the Higgs mass. For mixing with muon neutrinos,  $M_H < 267 \text{ GeV}$  (90% C.L.), with the best-fit value  $M_H = 136 \text{ GeV}$ . Conversely, such extra lepton singlets would be favoured with respect to the SM if the Higgs were eventually found to be heavy. We have also seen that an explanation of the NuTeV anomaly by the mixing of the SM neutrinos with extra neutrinos is precluded, in the absence of additional new physics, by the constraints imposed by other electroweak observables.

In Table 8, we collect the 90% C.L. bounds and the corresponding best values for the mixings between the different possible heavy vector-like leptons and the SM fermions. The mixing with the SM leptons can be as large as  $|V_L^{\tau N}| \sim 0.079$  at 90 % C.L. for heavy neutrino singlets mixing only with the third family. Other mixings are bounded to be less than  $\sim 0.06$  at 90 % C.L.. They are independent of the Dirac or Majorana character of the new leptons.

These limits have consequences for heavy lepton production and decay at large colliders. At LHC, they are in general more efficiently produced in pairs [31], except for heavy neutrino singlets, which have to be single produced in

association with SM leptons through their mixing, as they have no other SM interactions. In this case the new limits  $|V_L^{eN}| < 0.055$  and  $|V_L^{\mu N}| < 0.057$  are better than those found previously,  $|V_L^{eN}| < 0.074$  and  $|V_L^{\mu N}| < 0.098$  [19]. Therefore, the small parameter space which may be reached at the LHC [30] is further reduced. For instance, heavy Majorana neutrino singlets coupling only to muons may be observable at LHC for masses below 200 GeV. This limit can be much higher, however, in the presence of other interactions, up to 2 TeV for new right-handed gauge bosons of a similar mass and with a standard gauge coupling strength [45] (see for a review [46]). Dirac neutrino singlets are expected to be beyond the LHC reach. All other lepton additions can be pair produced, and then their discovery limit does not depend on the mixings, which only enter in the decay rates and are still large enough to allow the heavy leptons decay inside the detector. Hence, their rough discovery limit is near the TeV scale [31]. On the other hand, at  $e^+ e^-$  colliders the main production mechanism is through mixing with the first family. For instance, a neutrino singlet mixing with the electron neutrino with  $|V_L^{eN}| > 0.01$  is allowed by our bounds and would be observed at ILC for masses  $M_N < 400$  GeV, and at CLIC for  $M_N < 2$  TeV [34]. On the other hand, these stringent limits also makes more difficult the observation of possible deviations from unitarity in neutrino oscillations [47].

Vector-like leptons at the TeV scale appear naturally in many models, for example those with extra dimensions or larger gauge symmetries at low energy. As already emphasized, the new singlets and triplets of zero hypercharge can be Majorana and act as see-saw messengers of type I and III, respectively. If these fields exist with large mixings and relatively light masses, their contributions to neutrino masses and neutrinoless double  $\beta$  decay must be in general suppressed by extra, almost exact symmetries, typically LN [23, 26]. Thus, in general new leptons at the TeV scale and with relatively large mixings with the SM fermions must be (quasi)Dirac. If they are Majorana, the model must include a very efficient cancellation mechanism with an extended field content highly tuned [25].

The theory must also incorporate a rather precise alignment of the SM charged leptons and the new mass eigenstate leptons: each heavy lepton must mix mostly with only one light charged lepton to fulfill the limits on FCNC [20, 21, 22]. The corresponding limits are a factor 3 to 60 times more stringent than the flavour conserving ones, derived here. This justifies neglecting FCNC effects in our analysis, but also implies a strong constraint on definite models.

Finally, it is interesting to study how our conclusions would change in the presence of other new particles, which are actually present in many of the models mentioned above. We expect that the interference will be constructive in many cases. We have checked, for instance, that the new leptons can further improve the global fit of the extra-quark solution to the  $A_{\text{FB}}^{0,b}$  anomaly proposed in [10]. The effective formalism we used here is particularly convenient to perform fits involving many different kinds of new particles [16].

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## Appendix

Quantity		Experimental Value	Standard Model	Pull	Extended Model with $\Delta_1$ coupled to $\mu$	Pull
$m_t$	[6]	$172.6 \pm 1.4$	172.9	-0.2	172.9	-0.2
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$	[48]	$0.02758 \pm 0.00035$	0.02755	+0.1	0.02757	0.0
$\alpha_S(M_Z^2)$	[1]	$0.1176 \pm 0.002$	0.1181	-0.3	0.1176	0.0
$M_W$ [GeV]	[2]	$80.398 \pm 0.025$	80.365	+1.3	80.365	+1.3
$\text{Br}(W \rightarrow e\nu)$	[1]	$0.1075 \pm 0.0013$	0.1083	-0.6	0.1083	-0.6
$\text{Br}(W \rightarrow \mu\nu)$		$0.1057 \pm 0.0015$		-1.7		-1.7
$\text{Br}(W \rightarrow \tau\nu)$		$0.1125 \pm 0.0020$		+2.1		+2.1
$M_Z$ [GeV]	[49]	$91.1876 \pm 0.0021$	91.1876	0.0	91.1875	0.0
$\Gamma_Z$ [GeV]		$2.4952 \pm 0.0023$	2.4952	0.0	2.4947	+0.2
$\sigma_H^0$ [nb]		$41.541 \pm 0.037$	41.480	+1.7	41.489	+1.4
$R_e^0$		$20.804 \pm 0.050$	20.739	+1.3	20.735	+1.4
$R_\mu^0$		$20.785 \pm 0.033$	20.739	+1.4	20.781	+0.1
$R_\tau^0$		$20.764 \pm 0.045$	20.786	-0.5	20.782	-0.4
$A_{FB}^{0,e}$		$0.0145 \pm 0.0025$	0.0163	-0.7	0.163	-0.7
$A_{FB}^{0,\mu}$		$0.0169 \pm 0.0013$		+0.5	0.166	+0.3
$A_{FB}^{0,\tau}$		$0.0188 \pm 0.0017$		+1.5	0.163	+1.5
$A_e$ (SLD)	[49]	$0.1516 \pm 0.0021$	0.1474	+2.0	0.1474	+2.0
$A_\mu$ (SLD)		$0.142 \pm 0.015$		-0.4	0.1499	-0.5
$A_\tau$ (SLD)		$0.136 \pm 0.015$		-0.8	0.1474	-0.8
$A_e(P_\tau)$	[49]	$0.1498 \pm 0.0049$		+0.5	0.1474	+0.5
$A_\tau(P_\tau)$		$0.1439 \pm 0.0043$		-0.8	0.1474	-0.8
$R_b^0$	[49]	$0.21629 \pm 0.00066$	0.21581	+0.7	0.21581	+0.7
$R_c^0$		$0.1721 \pm 0.0030$	0.1722	0.0	0.1722	0.0
$A_{FB}^{0,b}$		$0.0992 \pm 0.0016$	0.1033	-2.6	0.1033	-2.6
$A_{FB}^{0,c}$		$0.0707 \pm 0.0035$	0.0738	-0.9	0.0738	-0.9
$A_b$		$0.923 \pm 0.020$	0.935	-0.6	0.935	-0.6
$A_c$		$0.670 \pm 0.027$	0.668	+0.1	0.668	+0.1
$\sin^2 \theta_{\text{eff}}^{\text{lept}}(Q_{\text{FB}}^{\text{had}})$	[49]	$0.2324 \pm 0.0012$	0.23148	+0.8	0.23148	+0.8
$g_L^2$	[1]	$0.3005 \pm 0.0012$	0.3038	-2.8	0.3038	-2.8
$g_R^2$		$0.0311 \pm 0.0010$	0.0301	+1.0	0.0301	+1.0
$\theta_L$		$2.51 \pm 0.033$	2.46	+1.4	2.46	+1.4
$\theta_R$		$4.59 \pm 0.41$	5.18	-1.4	5.18	-1.4
$g_V^{\nu e}$	[1]	$-0.040 \pm 0.015$	-0.0385	-0.1	-0.0384	-0.1
$g_A^{\nu e}$		$-0.507 \pm 0.014$	-0.5065	0.0	-0.5065	0.0
$Q_W(^{133}_{55}\text{Cs})$	[50]	$-72.74 \pm 0.46$	-72.92	+0.4	-72.92	+0.4

Table 11: Measurements of the (pseudo) observables included in our fit, compared with the best-fit values in the SM and in the SM extended by a  $\Delta_1$  doublet coupled to the second family.

Quantity		Experimental Value	Standard Model	Pull	Extended Model with $N$ Universal	Pull
$m_t$	[6]	$172.6 \pm 1.4$	172.9	-0.2	172.7	-0.1
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$	[48]	$0.02758 \pm 0.00035$	0.02756	+0.1	0.02769	-0.3
$\alpha_S(M_Z^2)$	[1]	$0.1176 \pm 0.002$	0.1181	-0.2	0.1181	-0.2
$M_W$ [GeV]	[2]	$80.398 \pm 0.025$	80.365	+1.3	80.362	+1.4
$\text{Br}(W \rightarrow l\nu)$	[1]	$0.1080 \pm 0.0009$	0.1083	-0.3	0.1082	-0.3
$M_Z$ [GeV]	[49]	$91.1875 \pm 0.0021$	91.1876	-0.1	91.1874	0.0
$\Gamma_Z$ [GeV]		$2.4952 \pm 0.0023$	2.4952	0.0	2.4961	-0.4
$\sigma_H^0$ [nb]		$41.540 \pm 0.037$	41.480	+1.6	41.501	+1.1
$R_l^0$		$20.767 \pm 0.025$	20.738	+1.2	20.740	+1.1
$A_{FB}^{0,l}$		$0.0171 \pm 0.0010$	0.0163	+0.8	0.0164	+0.7
$A_l$ (SLD)	[49]	$0.1513 \pm 0.0021$	0.1474	+1.9	0.1479	+1.6
$A_l(P_\tau)$	[49]	$0.1465 \pm 0.0033$		-0.3		-0.4
$R_b^0$	[49]	$0.21629 \pm 0.00066$	0.21582	+0.7	0.21582	+0.7
$R_c^0$		$0.1721 \pm 0.0030$	0.1722	0.0	0.1722	0.0
$A_{FB}^{0,b}$		$0.0992 \pm 0.0016$	0.1033	-2.6	0.1036	-2.8
$A_{FB}^{0,c}$		$0.0707 \pm 0.0035$	0.0738	-0.9	0.0741	-1.0
$A_b$		$0.923 \pm 0.020$	0.935	-0.6	0.935	-0.6
$A_c$		$0.670 \pm 0.027$	0.668	+0.1	0.668	+0.1
$\sin^2\theta_{\text{eff}}^{\text{lept}}(Q_{FB}^{\text{had}})$	[49]	$0.2324 \pm 0.0012$	0.23148	+0.8	0.23141	+0.8
$g_L^2$	[1]	$0.3005 \pm 0.0012$	0.3038	-2.8	0.3038	-2.8
$g_R^2$		$0.0311 \pm 0.0010$	0.0301	+1.0	0.0301	+1.0
$\theta_L$		$2.51 \pm 0.033$	2.46	+1.4	2.46	+1.4
$\theta_R$		$4.59 \pm 0.41$	5.18	-1.4	5.18	-1.4
$g_V^{\nu e}$	[1]	$-0.040 \pm 0.015$	-0.0384	-0.1	-0.0386	-0.1
$g_A^{\nu e}$		$-0.507 \pm 0.014$	-0.5065	0.0	-0.5064	0.0
$Q_W$ ( $^{133}\text{Cs}$ )	[50]	$-72.74 \pm 0.46$	-72.92	+0.4	-72.95	+0.5

Table 12: Measurements of the (pseudo) observables included in our fit assuming lepton universality, compared with the best-fit values in the SM and in the SM extended by universal singlets  $N$ .

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